Analysis of Melody Through Key Definition and Generation of Complementary Harmonies

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INTRODUCTION

Algorithmic music engages computational methods to create sound structures. Previous efforts in algorithmic music include automatic generation of music, sometimes through the application of biological models, e.g., genetic programming and genetic algorithms. Other notable methods use random walks and Markov chains.

Our focus is to create a mathematical model of theoretical notions found in common practice music. We provide a method of analysis that determines the key of an input melodic line by utilizing two different correlation metrics depending on consonance and scale membership, respectively. We then generate a harmonic progression that supports the melody within the determined key. To do this, we reuse the previous metrics with additional constraints such as harmonic relations with the given key and transitions between chords.

We test this model using prominent compositions from musical literature as input and noting how the output of the algorithm meets expectations or provides viable alternatives.

DETERMINING THE KEY OF AN INPUT SCORE

NOTE-COUNTING

We assume the musical work is sampled at some regular time interval. Our first step in analysis is to simply count up the number of occurrences of each pitch in the chromatic scale. We represent pitches on a chromatic scale as 12 values between 0 and 11, with 0 being C and 11 being B. For our purposes, notes that differ by octaves are considered equivalent and thus undergo a modulo 12 operation. We can thus define the **note-counting operation** on a pitch as the number of times it occurs in the piece.

CORRELATION FUNCTIONS FOR CHARACTERIZING KEYS

To measure the viability of a certain key with the given input score, we define two different correlation metrics with different criteria.

The **consonance correlation** emphasizes how euphonious a sequence of pitches is with respect to a certain key in accordance with common practice music. We first define the consonance of a note with respect to a given key in terms of the note's relation to the tonic of the key (see Table 1). The consonance correlation function for a key is then defined as the sum of the consonance relations for each pitch on the chromatic scale weighted by its appearance in the piece.

Relation of Note to Tonic	$consonance(note, key)$
1^{st}	1
5^{th}	$\frac{2}{3}$
4^{th}	$\frac{1}{3}$
3^{rd}, 6^{th} (same mode)	$\frac{1}{3}$
Other	0

Table 1: Consonance relations for a note with respect to a given key. For the 3rd and 6th, the note must be in the same mode as the key; else, the consonace relation is 0 (e.g, A^{b} with respect to C Major).

The **membership correlation** illustrates how well a pitch (or a sequence of pitches) fits into a given scale which is associated with a key. This is defined as the weighted sum of the characteristic function of that scale for all pitches that occur in the piece.

To find the key of the piece, we wish to maximize both of those correlation functions. However, a perfect membership correlation is not viable in common practice music as we often have the introduction of accidentals as well as temporary deviations into other keys. Thus, we settle on a *threshold value of 0.9 for the membership correlation*. The key of the piece is then chosen such that it maximizes the consonance correlation and would hence best characterize the piece.

A note about our approach: Our musical samples were from the beginnings of the pieces, which inherently reinforce the tonic by design and would be most telling in tonal music.

CREATING A COMPLEMENTARY HARMONIC PROGRESSION

We have obtained a significant amount of information now that we know the key of the piece. How can we use this information to generate a harmony that is plausible in the realm of common practice? We set forth a few reasonable approximations:

- We uniformly divide the piece into smaller subdivisions, usually representing beats oR measures.
- Each subdivision has one "key" or "chord" associated with it.
- We limit our choice of keys to (with respect to the key of the piece): the tonic, the dominant, the subdominant, and the relative minor/major (depending on the mode). Note that when we attempted to find the key of the entire piece, our key space could not be narrowed down because we had no other information. However, we can now reasonably limit our key space to those keys that are strongly associated with the key of the entire piece.

Using these assumptions, we simply apply the procedure for determining the key of a piece as before, except with our scope limited to a subdivision as input and the choice of keys as listed above. After doing this for all subdivisions, we have a list of keys which also represent a accompanying chord progression for the piece.

RESULTS AND DISCUSSION

KEY DEFINITION

We applied this method of key definition to samples of Western classical music, particularly those from the Baroque and Classical periods. Overall, we were able to successfully determine keys for 15 different pieces, which included the following:

- Bach: Invention No. 1 in C Major; Invention No. 2 in C minor; Minuet in G Major; Minuet in G minor
- Mozart: K. 545, 1^{st} mvt. (C Major) and 2^{nd} mvt. (G Major)
- Beethoven: Piano Sonata No. 1, 1^{st} mvt. (F minor); Piano Sonata No. 8 (Pathétique), 2^{nd} mvt. ($C^{\#}$ Major)

However, we observed limitations of our approach when considering Romantic pieces such as Chopin's Prelude in E minor, Op. 28 No. 4, where the melody consists of two notes. This suggests that in this work of Chopin, the harmony begins to play a more dominant role than the melody in the definition of a tonal space. It also suggests that the composer purposely exploited the tonal ambiguity present in such an extremely limited melodic pitch set as a means to develop his harmonic progressions.

Another interesting example was the Schumann Romance in $F^{\#}$ Major, Op. 28 No. 2, where the key returned was the relative minor of the actual key. However, the melody was harmonized on parallel thirds, and when the lower melody was accounted for, the proper key was returned. Thus, we see that as music progresses historically, it becomes difficult to precisely determine the key using common practice conventions. These contrary examples give us a much more realistic indication of the scope of our approach as well as the problem space.

HARMONY GENERATION

The output of a sequence of keys or chords for the harmonic progression leaves a great deal of flexibility for the form of the accompaniment. This is both a boon and a burden because while it does leave us with an open problem that can be attacked from multiple angles, we can explore each of these paths and the variations on the original melody-harmony relationship that they develop. Thus, we must consider the type of relation of the harmony with respect to the melody that we wish to emphasize. Some examples include:

- We must consider both the "vertical" support that a chord can provide a melody as well as the "horizontal" path that the chord progression lays out in terms of expressing the piece.
- A key for a chord leaves room for variation in inversions as well as placements in different octaves. This can result in parallel, contrary, and oblique movement as well as conjunct or disjunct motion.
- We also can raise the question of voicing since the melody is not necessarily assigned to a particular hand (if we consider the piano as our instrument) and since the analysis of melody is independent of placement within a certain octave.

Furthermore, we attempted to glean time information from the harmonic progression by analyzing a **return to tonic**, i.e., the length of time it takes until a the chord assigned to a subdivision is in the key of the entire piece. With pieces that were in the sonata form, for instance, we were able to see that this length was approximately twice the length of a measure. This roughly gives us the time signature in terms of subdivisions, which we can then use as an additional tool on shaping the harmony to be more applicable for the melody. We can improve our work here by implementing pattern recognition, e.g., accommodating for harmonic rhythm by being able to recognize cadences and other changes in tonal structure.

CONCLUSION

Our research goal was to produce a computational analysis technique for determining the key of an input melody and generate a complementary harmonic progression in accordance with common practice music. While we were able to see the success of our methods in many instances of Western classical music from the Baroque and Classical periods, we also saw the limitations in applying key definition to music from the Romantic period. Furthermore, our work in harmony generation, while relatively accurate, leaves many doors open when considering the actual form of the harmony, a fact that can be exploited in the future for variational purposes.

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